

# Non-Unitary and Unitary Transitions in Generalized Quantum Mechanics, New Small Parameter and Information Problem Solving

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## Abstract

Quantum Mechanics of the Early Universe is considered as deformation of a well-known Quantum Mechanics. Similar to previous works of the author, the principal approach is based on deformation of the density matrix with concurrent development of the wave function deformation in the respective Schrödinger picture, the associated deformation parameter being interpreted as a new small parameter. It is demonstrated that the existence of black holes in the suggested approach in the end twice causes nonunitary transitions resulting in the unitarity. In parallel this problem is considered in other terms: entropy density, Heisenberg algebra deformation terms, respective deformations of Statistical Mechanics, - all showing the identity of the basic results. From this an explicit solution for Hawking's information paradox has been derived.

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# 1 Introduction

As is known, the Early Universe Quantum Mechanics (Quantum Mechanics at Planck scale) is distinguished from a well-known Quantum Mechanics at conventional scales [1],[2] by the fact that in the first one the Generalized Uncertainty Relations (GUR) are fulfilled, resulting in the emergence of a fundamental length, whereas in the second one the usual Heisenberg Uncertainty Relations are the case. In case of Quantum Mechanics with Fundamental Length (QMFL) all three well-known fundamental constants are involved  $G, c, \hbar$ , while the classical QM is associated only with a single one  $\hbar$ . It is obvious that transition from the first to the second one within the inflation expansion is a nonunitary process, i.e. the process where the probabilities are not retained [3], [4]. Because of this, QMFL is considered as a deformation of QM. The deformation in Quantum Mechanics at Planck scale takes different paths: commutator deformation or more precisely deformation of the respective Heisenberg algebra [5],[6],[7], i.e. the density matrix deformation approach, developed by the author with co-workers in a number of papers [3],[4],[8],[9],[10]. The first approach suffers from two serious disadvantages: 1) the deformation parameter is a dimensional variable  $\kappa$  with a dimension of mass [5]; 2) in the limiting transition to QM this parameter goes to infinity and fluctuations of other values are hardly sensitive to it. Being devoid of the above limitation, the second approach by the author's opinion is intrinsic for QMFL: with it in QMFL the deformation parameter is represented by the dimensionless quantity  $\alpha = l_{min}^2/x^2$ , where  $x$  is the measuring scale and the variation interval  $\alpha$  is finite  $0 < \alpha \leq 1/4$  [3], [4],[10]. Besides, this approach contributes to the solution of particular problems such as the information paradox problem of black holes [3] and also the problem of an extra term in Liouville equation [8],[9],[10], derivation of Bekenstein-Hawking formula from the first principles [10], hypothesis of cosmic censorship [9],[10], more exact definition and expansion of the entropy notion through the introduction of the entropy density per minimum unit area [9],[10],[11]. Moreover, it is demonstrated that there exists a complete analogy in the construction and properties of quantum mechanics and statistical density matrices at Planck scale (density pro-matrices). It should be noted that an ordinary statistical density matrix appears in the low-temperature limit (at temperatures much lower than the Planck's)[12]. In the present work the unitarity problem for QMFL is considered on the basis of this approach. It is shown that as

distinct from Hawking's approach, in this treatment the existence of black holes is not the reason for the unitarity violation, rather being responsible for its recovery. First after the Big Bang (Original Singularity) expansion of the Universe is associated with the occurrence of a nonunitary transition from QMFL to QM, and with trapping of the matter by the black hole (Black Hole Singularity) we have a reverse nonunitary process from QM to QMFL. In such a manner a complete transition process from QMFL to the unitarity may be recovered. This fact may be verified differently by the introduction of a new value - entropy density matrix [11] with demonstration of the identity of entropy densities in the vicinity of the initial and final singularities. Thus, the existence of black holes contributes to the reconstruction of a symmetry of the general picture. Similar results may be obtained in terms of the Heisenberg's algebra deformation [5] and corresponding deformation of Statistical Mechanics [12],[20] associated with the Generalized Uncertainty Relations in Thermodynamics [20],[21],[22]. So the problem of Hawking information paradox is solved by the proposed approach: the information quantity in the Universe is preserved. In final sections of the paper it is demonstrated that  $\alpha$  interpreted as a new small parameter has particular advantages. This paper is a summing-up of the tentative results obtained by the author on the information paradox as an extension of the earlier works [3],[11] and [25].

## 2 Some Preliminary Facts

In this section the principal features of QMFL construction are briefly outlined first in terms of the density matrix deformation(von Neumann's picture) and subsequently in terms of the wave function deformation (Schrödinger picture) [3], [4],[9],[10]. As mentioned above, for the fundamental deformation parameter we use  $\alpha = l_{min}^2/x^2$ , where  $x$  is the measuring scale.

**Definition 1. (Quantum Mechanics with Fundamental Length [for Neumann's picture])**

Any system in QMFL is described by a density pro-matrix of the form

$$\rho(\alpha) = \sum_{\mathbf{i}} \omega_{\mathbf{i}}(\alpha) |\mathbf{i}\rangle \langle \mathbf{i}|,$$

where

1.  $0 < \alpha \leq 1/4$ ;
2. Vectors  $|i\rangle$  form a full orthonormal system;
3.  $\omega_i(\alpha) \geq 0$  and for all  $i$  the finite limit  $\lim_{\alpha \rightarrow 0} \omega_i(\alpha) = \omega_i$  exists;
4.  $Sp[\rho(\alpha)] = \sum_i \omega_i(\alpha) < 1$ ,  $\sum_i \omega_i = 1$ ;
5. For every operator  $B$  and any  $\alpha$  there is a mean operator  $B$  depending on  $\alpha$ :

$$\langle B \rangle_\alpha = \sum_i \omega_i(\alpha) \langle i | B | i \rangle.$$

should be fulfilled:

$$Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] \approx \alpha. \quad (1)$$

Consequently we can find the value for  $Sp[\rho(\alpha)]$  satisfying the condition of Definition 1:

$$Sp[\rho(\alpha)] \approx \frac{1}{2} + \sqrt{\frac{1}{4} - \alpha}. \quad (2)$$

According to point 5),  $\langle 1 \rangle_\alpha = Sp[\rho(\alpha)]$ . Therefore for any scalar quantity  $f$  we have  $\langle f \rangle_\alpha = f Sp[\rho(\alpha)]$ . We denote the limit  $\lim_{\alpha \rightarrow 0} \rho(\alpha) = \rho$  as the density matrix. Evidently, in the limit  $\alpha \rightarrow 0$  we return to QM.

In [3],[9],[10] it was shown that,

- I. the above limit covers both Quantum and Classical Mechanics.
- II. Density pro-matrix  $\rho(\alpha)$  tests singularities. As a matter of fact, the deformation parameter  $\alpha$  should assume value  $0 < \alpha \leq 1$ . However, as seen from (2),  $Sp[\rho(\alpha)]$  is well defined only for  $0 < \alpha \leq 1/4$ , i.e. for  $x = il_{min}$  and  $i \geq 2$  we have no problems at all. At the point, where  $x = l_{min}$  (that corresponds to a singularity of space),  $Sp[\rho(\alpha)]$  takes the complex values.
- III. It is possible to read equation (1) as

$$Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] = \alpha + a_0 \alpha^2 + a_1 \alpha^3 + \dots \quad (3)$$

Then for example, one of the solutions of (1) is

$$\rho^*(\alpha) = \sum_i \alpha_i \exp(-\alpha) |i\rangle \langle i|, \quad (4)$$

where all  $\alpha_i > 0$  are independent of  $\alpha$  and their sum is equal to 1. In this way  $Sp[\rho^*(\alpha)] = \exp(-\alpha)$ . Note that in the momentum representation  $\alpha = p^2/p_{max}^2$ ,  $p_{max} \sim p_{pl}$ , where  $p_{pl}$  is the Planck momentum. When present in matrix elements,  $\exp(-\alpha)$  can damp the contribution of great momenta in a perturbation theory. The solution (1) given by the formula (4) is further referred to as (**exponential ansatz**). This ansatz will be the principal one in our further consideration.

In [9], [10] it has been demonstrated, how a transition from Neumann's picture to Shrödinger's picture, i.e. from the density matrix deformation to the wave function deformation, may be realized by the proposed approach

**Definition 2. (Quantum Mechanics with Fundamental Length [Shrödinger's picture])**

Here, the prototype of Quantum Mechanical normed wave function (or the pure state prototype)  $\psi(q)$  with  $\int |\psi(q)|^2 dq = 1$  in QMFL is  $\theta(\alpha)\psi(q)$ . The parameter of deformation  $\alpha$  assumes the value  $0 < \alpha \leq 1/4$ . Its properties are  $|\theta(\alpha)|^2 < 1$ ,  $\lim_{\alpha \rightarrow 0} |\theta(\alpha)|^2 = 1$  and the relation  $|\theta(\alpha)|^2 - |\theta(\alpha)|^4 \approx \alpha$  takes place. In such a way the total probability always is less than 1:  $p(\alpha) = |\theta(\alpha)|^2 = \int |\theta(\alpha)|^2 |\psi(q)|^2 dq < 1$  tending to 1 when  $\alpha \rightarrow 0$ . In the most general case of the arbitrarily normed state in QMFL (mixed state prototype)  $\psi = \psi(\alpha, q) = \sum_n a_n \theta_n(\alpha) \psi_n(q)$  with  $\sum_n |a_n|^2 = 1$  the total probability is  $p(\alpha) = \sum_n |a_n|^2 |\theta_n(\alpha)|^2 < 1$  and  $\lim_{\alpha \rightarrow 0} p(\alpha) = 1$ .

It is natural that Shrödinger equation is also deformed in QMFL. It is replaced by the equation

$$\frac{\partial \psi(\alpha, q)}{\partial t} = \frac{\partial [\theta(\alpha)\psi(q)]}{\partial t} = \frac{\partial \theta(\alpha)}{\partial t} \psi(q) + \theta(\alpha) \frac{\partial \psi(q)}{\partial t}, \quad (5)$$

where the second term in the right-hand side generates the Shrödinger equation as

$$\theta(\alpha) \frac{\partial \psi(q)}{\partial t} = \frac{-i\theta(\alpha)}{\hbar} H \psi(q). \quad (6)$$

Here  $H$  is the Hamiltonian and the first member is added similarly to the member that appears in the deformed Liouville equation, vanishing

when  $\theta[\alpha(t)] \approx const$ . In particular, this takes place in the low energy limit in QM, when  $\alpha \rightarrow 0$ . It should be noted that the above theory is not a time reversal of QM because the combination  $\theta(\alpha)\psi(q)$  breaks down this property in the deformed Shrödinger equation. Time-reversal is conserved only in the low energy limit, when a quantum mechanical Shrödinger equation is valid.

### 3 Some Comments and Unitarity in QMFL

As has been indicated in the previous section, time reversal is retained in the large-scale limit only. The same is true for the superposition principle in Quantum Mechanics. Indeed, it may be retained in a very narrow interval of cases for the functions  $\psi_1(\alpha, q) = \theta(\alpha)\psi_1(q)$   $\psi_2(\alpha, q) = \theta(\alpha)\psi_2(q)$  with the same value  $\theta(\alpha)$ . However, as for all  $\theta(\alpha)$ , their limit is  $\lim_{\alpha \rightarrow 0} |\theta(\alpha)|^2 = 1$  or equivalently  $\lim_{\alpha \rightarrow 0} |\theta(\alpha)| = 1$ , in going to the low-energy limit each wave function  $\psi(q)$  is simply multiplied by the phase factor  $\theta(0)$ . As a result we have Hilbert Space wave functions in QM. Comparison of both pictures (Neumann's and Shrödinger's) is indicative of the fact that unitarity means the retention of the probabilities  $\omega_i(\alpha)$  or retention of the squared modulus (and hence the modulus) for the function  $\theta(\alpha)$ :  $|\theta(\alpha)|^2, (|\theta(\alpha)|)$ . That is

$$\frac{d\omega_i[\alpha(t)]}{dt} = 0$$

or

$$\frac{d|\theta[\alpha(t)]|}{dt} = 0.$$

In this way a set of unitary transformations of QMFL includes a group  $U$  of the unitary transformations for the wave functions  $\psi(q)$  in QM. It is seen that on going from Planck's scale to the conventional one, i.e. on transition from the Early Universe to the current one, the scale has been rapidly changing in the process of inflation expansion and the above conditions failed to be fulfilled:

$$\frac{d\omega_i[\alpha(t)]}{dt} \neq 0, \frac{d|\theta[\alpha(t)]|}{dt} \neq 0. \quad (7)$$

In terms of the density pro-matrices of section 2 this is a limiting transition from the density pro-matrix in QMFL  $\rho(\alpha), \alpha > 0$ , that is a prototype of the pure state at  $\alpha \rightarrow 0$ , to the density matrix  $\rho(0) = \rho$  representing a pure state in QM. Mathematically this means that a nontotal probability (below 1) is changed by the total one (equal to 1). For the wave functions in Schrödinger picture this limiting transition from QMFL to QM is as follows:

$$\lim_{\alpha \rightarrow 0} \theta(\alpha) \psi(q) = \psi(q)$$

up to the phase factor.

It is apparent that the above transition from QMFL to QM is not a unitary process, as indicated in [3],[4],[8]-[10]. However, the unitarity may be recovered when we consider in a sense a reverse process: absorption of the matter by a black hole and its transition to singularity conforming to the reverse and nonunitary transition from QM to QMFL. Thus, nonunitary transitions occur in this picture twice:

$$I.(QMFL, OS, \alpha \approx 1/4) \xrightarrow{\text{Big Bang}} (QM, \alpha \approx 0)$$

$$II.(QM, \alpha \approx 0) \xrightarrow{\text{absorbing}} (QMFL, SBH, \alpha \approx 1/4),$$

Here the following abbreviations are used: OS for the Origin Singularity; BH for a Black Hole; SBH for the Singularity in Black Hole.

As a result of these two nonunitary transitions, the total unitarity may be recovered:

$$III.(QMFL, OS, \alpha \approx 1/4) \longrightarrow (QMFL, SBH, \alpha \approx 1/4)$$

In such a manner the total information quantity in the Universe remains

unchanged, i.e. no information loss occurs.

In terms of the deformed Liouville equation [8]-[10] we arrive to the expression with the same right-hand parts for  $t_{initial} \sim t_{Planck}$  and  $t_{final}$  (for  $\alpha \approx 1/4$ ).

$$\frac{d\rho[\alpha(t), t]}{dt} = \sum_i \frac{d\omega_i[\alpha(t)]}{dt} |i(t)\rangle \langle i(t)| - \\ -i[H, \rho(\alpha)] = d[\ln\omega(\alpha)]\rho(\alpha) - i[H, \rho(\alpha)]. \quad (8)$$

It should be noted that for the closed Universe one can consider Final Singularity (FS) rather than the Singularity of Black Hole (SBH), and then the right-hand parts of diagrams II and III will be changed:

$$IIa.(QM, \alpha \approx 0) \xrightarrow{\text{Big Bang}} (QMFL, FS, \alpha \approx 1/4),$$

$$IIIa.(QMFL, OS, \alpha \approx 1/4) \longrightarrow (QMFL, FS, \alpha \approx 1/4)$$

At the same time, in this case the general unitarity and information are still retained, i.e. we again have the unitary product of two nonunitary arrows:

$$IV.(QMFL, OS, \alpha \approx 1/4) \xrightarrow{\text{Big Bang}} (QM, \alpha \approx 0) \xrightarrow{\text{Big Crunch}} (QMFL, FS, \alpha \approx 1/4),$$

Finally, arrow III may appear immediately, i.e. without the appearance of arrows I II, when in the Early Universe mini BH are arising:

$$IIIb.(QMFL, OS, \alpha \approx 1/4) \longrightarrow (QMFL, mini BH, SBH, \alpha \approx 1/4)$$

Note that here, unlike the previous cases, a unitary transition occurs immediately, without any additional nonunitary ones, and with retention of the total information.

Another approach to the information paradox problem associated with the above-mentioned methods (density matrix deformation) is the introduction and investigation of a new value namely: entropy density per minimum unit area. This approach is described in section 4.

## 4 Entropy Density Matrix and Information Loss Problem

In [3],[4],[8],[9],[10] the authors were too careful, when introducing for density pro-matrix  $\rho(\alpha)$  the value  $S_\alpha$  generalizing the ordinary statistical entropy:

$$S_\alpha = -Sp[\rho(\alpha) \ln(\rho(\alpha))] = - < \ln(\rho(\alpha)) >_\alpha .$$

In [9],[10] it was noted that  $S_\alpha$  means the entropy density on a minimum unit area depending on the scale. In fact a more general concept accepts the form of the entropy density matrix [11]:

$$S_{\alpha_2}^{\alpha_1} = -Sp[\rho(\alpha_1) \ln(\rho(\alpha_2))] = - < \ln(\rho(\alpha_2)) >_{\alpha_1}, \quad (9)$$

where  $0 < \alpha_1, \alpha_2 \leq 1/4$ .

$S_{\alpha_2}^{\alpha_1}$  has a clear physical meaning: the entropy density is computed on the scale associated with the deformation parameter  $\alpha_2$  by the observer who is at a scale corresponding to the deformation parameter  $\alpha_1$ . Note that with this approach the diagonal element  $S_\alpha = S_\alpha^\alpha$ , of the described matrix  $S_{\alpha_2}^{\alpha_1}$  is the density of entropy measured by the observer who is at the same scale as the measured object associated with the deformation parameter  $\alpha$ . In [10] section 6 such a construction was used implicitly in derivation of the semiclassical Bekenstein-Hawking formula for the Black Hole entropy:

a) For the initial (approximately pure) state

$$S_{in} = S_0^0 = 0$$

b) Using the exponential ansatz(4),we obtain:

$$S_{out} = S_{\frac{1}{4}}^0 = - \langle \ln[\exp(-1/4)]\rho_{pure} \rangle = - \langle \ln(\rho(1/4)) \rangle = \frac{1}{4}.$$

So increase in the entropy density for an external observer at the large-scale limit is  $1/4$ . Note that increase of the entropy density(information loss) for the observer crossing the horizon of the black hole's events and moving with the information flow to singularity will be smaller:

$$S_{out} = S_{\frac{1}{4}}^{\frac{1}{4}} = -Sp(\exp(-1/4)\ln[\exp(-1/4)]\rho_{pure}) = - \langle \ln(\rho(1/4)) \rangle_{\frac{1}{4}} \approx 0.1947.$$

It is clear that this fact may be interpreted as follows: for the observer moving together with information its loss can occur only at the transition to smaller scales, i.e. to greater deformation parameter  $\alpha$ .

Now we consider the general Information Problem. Note that with the classical Quantum Mechanics (QM) the entropy density matrix  $S_{\alpha_2}^{\alpha_1}$  (9) is reduced only to one element  $S_0^0$ . So we can not test anything. Moreover, in previous works relating the quantum mechanics of black holes and information paradox [16],[17, 18] the initial and final states when a particle hits the hole are treated proceeding from different theories (QM and QMFL respectively), as was indicated in diagram II:

(Large-scale limit, QM, density matrix) → (Black Hole, singularity, QMFL, density pro-matrix),

Of course in this case any conservation of information is impossible as these theories are based on different concepts of entropy. Simply saying, it is incorrect to compare the entropy interpretations of two different theories (QM and QMFL)where this notion is originally differently understood. So the chain above must be symmetrized by accompaniment of the arrow on the left ,so in an ordinary situation we have a chain (diagram III):

(Early Universe, origin singularity, QMFL, density pro-matrix) →  
 (Large-scale limit, QM, density matrix) → (Black Hole, singularity, QMFL, density pro-matrix),

So it's more correct to compare entropy close to the origin and final (Black hole) singularities. In other words, it is necessary to take into account

not only the state, where information disappears, but also that whence it appears. The question arises, whether in this case the information is lost for every separate observer. For the event under consideration this question sounds as follows: are the entropy densities  $S(\text{in})$  and  $S(\text{out})$  equal for every separate observer? It will be shown that in all conceivable cases they are equal.

1) For the observer in the large-scale limit (producing measurements in the semiclassical approximation)  $\alpha_1 = 0$

$$S(\text{in}) = S_{\frac{1}{4}}^0 \text{ (Origin singularity)}$$

$$S(\text{out}) = S_{\frac{1}{4}}^0 \text{ (Singularity in Black Hole)}$$

So  $S(\text{in}) = S(\text{out}) = S_{\frac{1}{4}}^0$ . Consequently, the initial and final densities of entropy are equal and there is no information loss.

2) For the observer moving together with the information flow in the general situation we have the chain:

$$S(\text{in}) \rightarrow S(\text{large-scale}) \rightarrow S(\text{out}),$$

where  $S(\text{large-scale}) = S_0^0 = S$ . Here  $S$  is an ordinary entropy of Quantum Mechanics(QM), but  $S(\text{in}) = S(\text{out}) = S_{\frac{1}{4}}^{\frac{1}{4}}$ , - value considered in QMFL. So in this case the initial and final densities of entropy are equal without any loss of information.

3) This case is a special case of 2), when we do not come out of the Early Universe considering the processes with the participation of black mini-holes only. In this case the originally specified chain becomes shorter by one section (diagram IIIb):

(Early Universe, origin singularity, QMFL, density pro-matrix)  $\rightarrow$  (Black Mini-Hole, singularity, QMFL, density pro-matrix),

and member  $S(\text{large-scale}) = S_0^0 = S$  disappears at the corresponding chain of the entropy density associated with the large-scale consideration:

$$S(\text{in}) \rightarrow S(\text{out}),$$

It is, however, obvious that in case  $S(in) = S(out) = S_{\frac{1}{4}}^{\frac{1}{4}}$  the density of entropy is preserved. Actually this event was mentioned in section 5 [10], where from the basic principles it has been found that black mini-holes do not radiate, just in agreement with the results of other authors [13]-[15],[19]

As a result, it's possible to write briefly

$$S(in) = S(out) = S_{\frac{1}{4}}^{\alpha},$$

where  $\alpha$  - any value in the interval  $0 < \alpha \leq 1/4$ .

Actually our inferences are similar to those of previous section in terms of the Liouville's equation deformation (8):

$$\frac{d\rho}{dt} = \sum_i \frac{d\omega_i[\alpha(t)]}{dt} |i(t)><i(t)| - i[H, \rho(\alpha)] = d[ln\omega(\alpha)]\rho(\alpha) - i[H, \rho(\alpha)].$$

The main result of this section is a necessity to account for the member  $d[ln\omega(\alpha)]\rho(\alpha)$ , deforming the right-side expression of  $\alpha \approx 1/4$ .

## 5 Unitarity, Non-Unitarity and Heisenbergs Algebra Deformation

The above-mentioned unitary and nonunitary transitions may be described in terms of Heisenbergs algebra deformation (deformation of commutators) as well. We use the principal results and designations from [5]. In the process the following assumptions are resultant: 1) The three-dimensional rotation group is not deformed; angular momentum  $\mathbf{J}$  satisfies the undeformed  $SU(2)$  commutation relations, whereas the coordinate and momenta satisfy the undeformed commutation relations  $[J_i, x_j] = i\epsilon_{ijk}x_k, [J_i, p_j] = i\epsilon_{ijk}p_k$ . 2) The momenta commute between themselves:  $[p_i, p_j] = 0$ , so the translation group is also not deformed. 3) Commutators  $[x, x]$  and  $[x, p]$  depend on the deformation parameter  $\kappa$  with the dimension of mass. In the limit  $\kappa \rightarrow \infty$  with  $\kappa$  much larger than any energy the canonical commutation relations are recovered.

For a specific realization of points 1) to 3) the generating GUR are of the

form [5]: ( $\kappa$ -deformed Heisenberg algebra)

$$[x_i, x_j] = -\frac{\hbar^2}{\kappa^2} i\epsilon_{ijk} J_k \quad (10)$$

$$[x_i, p_j] = i\hbar\delta_{ij}(1 + \frac{E^2}{\kappa^2})^{1/2}. \quad (11)$$

Here  $E^2 = p^2 + m^2$ . Note that in this formalism the transition from GUR to UR, or equally from QMFL to QM with  $\kappa \rightarrow \infty$  or from Planck scale to the conventional one, is nonunitary exactly following the transition from density pro-matrix to the density matrix in previous sections:

$$\rho(\alpha \neq 0) \xrightarrow{\alpha \rightarrow 0} \rho$$

Then the first arrow I in the formalism of this section may be as follows:

$$I'.(GUR, OS, \kappa \sim M_p) \xrightarrow{\text{Big Bang}} (UR, \kappa = \infty)$$

or what is the same

$$I''.(QMFL, OS, \kappa \sim M_p) \xrightarrow{\text{Big Bang}} (QM, \kappa = \infty),$$

where  $M_p$  is the Planck mass. In some works of the last two years Quantum Mechanics for a Black Hole has been already considered as a Quantum Mechanics with GUR [13]-[15]. As a consequence, by this approach the Black Hole is not completely evaporated but rather some stable remnants always remain in the process of its evaporation with a mass  $\sim M_p$ . In terms of [5] this means nothing else but a reverse transition:  $(\kappa = \infty) \rightarrow (\kappa \sim M_p)$ . And for an outside observer this transition is of the form:

$$II'.(UR, \kappa = \infty) \xrightarrow{\text{absorbing BH}} (GUR, SBH, \kappa \sim M_p),$$

$$II''.(QM, \kappa = \infty) \xrightarrow{\text{absorbing BH}} (QMFL, SBH, \kappa \sim M_p).$$

So similar to the previous section, two nonunitary inverse transitions a)  $I'$ , ( $I''$ ) and b)  $II'$ , ( $II''$ ) are liable to generate a unitary transition:

$$III'.(GUR, OS, \kappa \sim M_p) \xrightarrow{\text{Big Bang}} (UR, \kappa = \infty) \xrightarrow{\text{absorbing BH}} (GUR, SBH, \kappa \sim M_p),$$

or to summarize

$$III''.(GUR, OS, \kappa \sim M_p) \rightarrow (GUR, SBH, \kappa \sim M_p)$$

In conclusion of this section it should be noted that an approach to the Quantum Mechanics at Planck Scale using the Heisenberg algebra deformation (similar to the approach based on the density matrix deformation from the previous section) gives a deeper insight into the possibility of retaining the unitarity and the total quantity of information in the Universe, making possible the solution of Hawking's information paradox problem [16]-[18].

## 6 Statistical Mechanics Deformation and Transitions

Naturally, deformation of Quantum Mechanics in the Early Universe is associated with the Statistical Mechanics deformation as indicated in [12],[20]. In case under consideration this simply implies a transition from the Generalized Uncertainty Relations (GUR) of Quantum Mechanics to GUR in Thermodynamics [20]- [22]. The latter are distinguished from the normal uncertainty relations by:

$$\Delta \frac{1}{T} \geq \frac{k}{\Delta U} \quad (12)$$

i.e. by inclusion of the high-temperature term into the right-hand side

$$\Delta \frac{1}{T} \geq \frac{k}{\Delta U} + \alpha' \frac{1}{T_p^2} \frac{\Delta U}{k} + \dots \quad (13)$$

dots meaning the existence of higher order corrections [20]. Thus, denoting the Generalized Uncertainty Relations in Thermodynamics as GURT and using abbreviation URT for the conventional ones, we obtain a new form of diagram I from section III ( $I'$  of section IV respectively):

$$I^T.(GURT, OS) \xrightarrow{\text{Big Bang}} (URT)$$

In [12],[20] the Statistical Mechanics deformation associated with GURT

is described by the introduction of the respective deformation for the statistical density matrix  $\rho_{stat}(\tau)$  where  $0 < \tau \leq 1/4$ . Obviously, close to the Origin Singularity  $\tau \approx 1/4$ . Because of this, arrow  $I^T$  may be represented in a more general form as

$$I^{Stat}.(GURT, OS, \rho_{stat}(\tau), \tau \approx 1/4) \xrightarrow{\text{Big Bang}} (URT, \rho_{stat}, \tau \approx 0)$$

The reverse transition is also possible. In [13]-[15] it has been shown that Statistical Mechanics of Black Hole should be consistent with the deformation of a well-known Statistical Mechanics. The demonstration of an \*upper\* bound for temperature in Nature, given by Planck temperature and related to Black Hole evaporation, was provided in [23]. It is clear that Emergence of such a high temperatures is due to GURT. And we have the following diagram that is an analog of diagrams II and  $II'$  for Statistical Mechanics:

$$II^{Stat}.(URT, \rho_{stat}, \tau \approx 0) \xrightarrow{\text{absorbing}} {}^{BH} (GURT, SBH, \rho_{stat}(\tau), \tau \approx 1/4).$$

By this means, combining  $I^{Stat}$  and  $II^{Stat}$ , we obtain  $III^{Stat}$  representing a complete statistical-mechanics analog for quantum-mechanics diagrams  $III$  and  $III'$ :

$$III^{Stat}.(GURT, OS, \tau \approx 1/4) \xrightarrow{\text{Big Bang, absorbing}} {}^{BH} (GURT, SBH, \tau \approx 1/4).$$

And in this case two nonunitary transitions  $I^{Stat}$  and  $II^{Stat}$  in the end lead to a unitary transition  $III^{Stat}$ .

## 7 Measuring Procedure and New Small Parameter

As noted above, the primary relation may be written in the form of a series

$$Sp[\rho(\alpha)] - Sp^2[\rho(\alpha)] = \alpha + a_0\alpha^2 + a_1\alpha^3 + \dots \quad (14)$$

As a result, a measurement procedure using the exponential ansatz (4) may be understood as the calculation of factors  $a_0, a_1, \dots$  or the definition of additional members in the exponent destroying  $a_0, a_1, \dots$ . It is easy to

check that the exponential ansatz gives  $a_0 = -3/2$ , being coincident with the logarithmic correction factor for the Black Hole entropy [24].

From section 2 and specifically from relation (14) it follows that  $\alpha$  is a new small parameter. Among its obvious advantages one could name:

1) its dimensionless nature,

2) its variability over the finite interval  $0 < \alpha \leq 1/4$ . Besides, for the well-known physics it is actually very small:  $\alpha \sim 10^{-66+2n}$ , where  $10^{-n}$  is the measuring scale. Here the Planck scale  $\sim 10^{-33} cm$  is assumed;

3) and finally the calculation of this parameter involves all three fundamental constants, since by Definition 1 of section 2  $\alpha = l_{min}^2/x^2$ , where  $x$  is the measuring scale and  $l_{min}^2 \sim l_{pl}^2 = G\hbar/c^3$ .

Therefore, series expansion in  $\alpha$  may be of great importance. Especially as all field constants of any quantum system by Definition 2 of section 2 are dependent on  $\alpha$ , i.e.  $\psi = \psi(\alpha)$ .

## 8 Conclusion

Thus, this work outlines that the existence of GUR and hence the appearance of QMFL enables a better understanding of the information problem in the Universe providing a key to the solution of this problem in a not inconsistent manner, practically in the same way but irrespective of the approach used: 1) density matrix deformation in Quantum (Statistical) Mechanics at Planck's scale (and as a consequence, entropy density matrix approach) or 2) Heisenberg algebra deformation.

It should be noted that the question of the relationship between these two approaches, i.e. transition from one deformation to the other, still remains open. This aspect is to be studied in further investigations of the author.

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